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NASA TT F-8352
Code-2d
X63-11435

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# CONTRIBUTION TO THE THEORY OF LOW-FREQUENCY GRAVITATIONAL WAVES


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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON February 1963

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# CONTRIBUTION TO THE DETECTION OF LOW-FREQUENCY GRAVITATIONAL WAVES

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It is shown that the sensitivity of electromechanical experiments employing piezocrystals to detect gravitational waves is ten orders less than estimated by Weber [1]. In the low-frequency range it is possible to detect gravitational waves by the shift in the bands of an optical interferometer. The sensitivity of this method is evaluated. auth. abstr.

The problem of the detection of gravitational waves has recently become a subject of discussion in the literature [1,2], with the stress being placed on electromechanical experiments. Nevertheless, the nonrelativistic bodies at the disposal of the experimenter interact very weakly with gravitational waves. Let us consider the equation of motion of a particle in the nonrelativistic approximation in the presence of an external electromagnetic field $F^{ik}$ [3]: /605

$$mc\left[\frac{du^{\alpha}}{ds} + \Gamma^{\alpha}_{ik}u^{i}u^{k}\right] = \frac{e}{c}F^{\alpha}{}_{k}u^{k} = \frac{e}{c}F^{\alpha k}u^{k} + \frac{e}{c}F^{\alpha s}h_{sk}u^{k}. \qquad (1)$$

A plane gravitational wave ($g_{00} = -1, g_{0\alpha} = 0$) does not affect the proper time of a nonrelativistic ($u^{\alpha} = 0$) body, $\Gamma^{\alpha}_{00} = 0$; an uncharged nonrelativistic particle does not receive the wave. If the field $F^{ik}$ is created by given nonrelativistic charges and currents, unaffected by the action of an approaching gravitational wave, then the field $F^{ik}$ itself will not be affected, as is directly evident from the equations of the field [3].

$$F^{ik}{}_{;k} = \partial F^{ik}/\partial x^k = (4\pi/c)\, j^i \qquad (2)$$

since in the gravitational wave $\sqrt{-g} = 1$. The presence of a gravitational wave leads to the appearance of an additional force on the right-hand side of (1), $eF^{\alpha s} h_{sk} u^k$, which vanishes in the nonrelativistic approximation ($k = 0$). The reception of gravitational waves by a nonrelativistic body (including one of the piezoelectric type) is comparatively ineffective.

Starting from general propositions relating to linear processes[1], we shall show that the sensitivity given by Weber [1] is several orders too high. The limiting sensitivity of experiments designed to detect weak gravitational waves is determined by the linear processes that predominate with weak fields. If the equations of an arbitrary linear system are reversible in time, there is a relation linking the energy losses due to radiation and the effective diameter $\sigma$ in reception:

$$\sigma = \sigma_o G\eta, \quad \eta = Q_o/Q_R, \qquad (3)$$

where $\sigma_o$ = effective cross section of the ideal lossless antenna, $G$ = amplification factor due to directivity, $\eta$ = efficiency of the antenna in transmission, $Q_o$ = Q factor of the real antenna, $Q_R$ = Q factor connected with radiation [1]. The cross section $\sigma_o \cong \lambda^2$ correct to a factor of the order of unity. An expression of type (3), familiar in antenna theory [4], can be obtained from Weber's formulas [1] and also follows from the principle of detailed balance /606

1 It is known that a rotating shaft or a binary star is characterized by quadripolar emission, the frequency of the radiation being twice the frequency of motion in the system. These processes, in which the frequency is doubled, are not linear and are not discussed here.

(see [5], section 117). A check shows that Weber's results do not satisfy expression (3), the discrepancy being of the order of $10^{10}$. Thus, for example, according to Weber [1], for a wave with $\lambda = 100$ cm, under radiating conditions, the power supplied to the crystal is $10^{8}$ watts, and the radiated power $10^{-13}$ ergs/sec, whence $\eta = 10^{-28}$, the radiation is quadripolar, and G = 15. Under receiving conditions, for the ideal antenna $\sigma_0 = 3 \cdot 10^{3}$ cm$^2$, while for the real antenna formula (3) gives $\sigma = 4 \cdot 10^{-24}$ cm$^2$. With Weber's threshold value for the energy flux $P = 10^{-3}$ ergs/sec$\cdot$cm$^2$, the electromagnetic power received is $4 \cdot 10^{-34}$ watts =-334 db/watt, which is 110 db below the threshold power for an ideal receiver with a noise temperature of 3°K and a 1-cycle band. The time required to detect a signal 100 db below the threshold would exceed $10^4$ years.

Weber calculated the radiated power from the "quadripole" formula [3], which there is no reason to doubt. Therefore, it follows from (3) that he made an error in calculating the power received. The cause of his error lies in the fact that in the piezocrystal the piezoelectric strains are counterbalanced by mechanical ones, a factor Weber did not take into account. The piezoelectric strains do not satisfy the virial theorem [3]. Since the reception of gravitational waves is a relativistic effect, it is to be expected that the use of an ultrarelativistic body -- light -- might lead to a more efficient indicator of the field of a gravitational wave. The optics of rays in a gravitational field are determined by the eikonal equation [3]:

$$g^{ik} \frac{\partial\psi}{\partial x^i} \frac{\partial\psi}{\partial x^k} = \left(\frac{\partial\psi}{\partial x^i}\right)^2 - h^{\alpha\beta} \left(\frac{\partial\psi}{\partial x^\alpha}\right)\left(\frac{\partial\psi}{\partial x^\beta}\right) = 0, \qquad (4)$$

where $\psi$ is the eikonal. This is equivalent to a medium with a refractive index

$$n = 1 + \frac{1}{2} h_{\alpha\beta} n^\alpha n^\beta, \qquad (5)$$

where $n^{\alpha}$ is the unit vector in the direction of propagation of the ray. For the propagation of a ray along and across a gravitational wave we have

$$n_{\parallel} = 1, \quad n_{\perp} = 1 + \frac{1}{2} h_{22} \cos 2\varphi + \frac{1}{2} h_{23} \sin 2\varphi, \quad \cos\varphi = n_2. \tag{6}$$

In an instrument of the Michelson interferometer type the relative difference in the optical lengths of light rays traveling along and across a gravitational wave will be

$$\Delta l / l_0 = \frac{1}{2} h_{\alpha\beta} n^{\alpha} n^{\beta}, \tag{7}$$

where $l_0$ is the undisturbed length of the arm of the interferometer. Note that formula (7) for the Michelson interferometer can also be obtained directly. In the gravitational wave the optical length of the arm of the interferometer changes, and the relative difference ([3], section 84) will be

$$\frac{\Delta l}{l_0} = \frac{1}{l_0} \int_0^{l_0} \sqrt{g_{22}}\, dx_2 - \frac{1}{l_0} \int_0^{l_0} \sqrt{g_{11}}\, dx_1 \approx \frac{1}{2} h_{22}. \tag{7a}$$

In deriving formula (7) it was assumed that the period of the gravitational wave was considerably greater than the transit time of the ray in the interferometer.

Thus, a gravitational wave will produce a periodic shift in the interference bands. Let us express (7) in terms of the flux of /607
radiation energy P, first turning the axes $Ox_2$ and $Ox_3$ so as to suppress the component $h_{23}$. In this system only $h_{22} = -h_{33} = h$ will be other than zero, and accordingly the flux of gravitational energy will be

$$P = \omega^2 c^3 h^2 \,/\, 16\pi k = \omega^2 c h^2 \,/\, 2\kappa ; \; \kappa = 8\pi k/c^2 \,, \qquad (8)$$

where $\kappa$ is Einstein's gravitational constant. Making use of (7), we have

$$\frac{\Delta l}{l_o} = \frac{1}{2\omega}\sqrt{\frac{2\kappa P}{c}} = \frac{8.1\cdot 10^{-20}}{f(\mathrm{cps})}\sqrt{P\ (\mathrm{ergs/cm^2\cdot sec})}. \qquad (9)$$

The minimum $\Delta l$ measured with ordinary light sources [6,7] are $10^{-3}$ Å, or $10^{-11}$ cm, for instruments with a time constant $\tau$ ~1sec. It is to be expected that the use of strong sources and amplifiers of monochromatic directional light emission -- lasers [8] would reduce this value by a further three orders.

Taking an interferometer arm $l_o \approx 10^3$ cm, for the minimum detectable variation we get $\Delta l/l_o \approx 10^{-14} \div 10^{-7}$; $\tau \sim 1$ sec.

Thus, at least in principle, the interferometer makes it possible to detect very weak gravitational waves. When $f_o = 10^{-3}$ cycles, $P = 1$ erg/cm$^2\cdot$sec. $\Delta l/l \cong 8\cdot 10^{-17}$, which is roughly $10^7 \div 10^{10}$ times better than in electromechanical experiments [1].

A further gain in sensitivity is possible by increasing the observation time and making use of known methods of separating a weak signal above the noise level. Evidently, real observation times $\tau \sim 10^4$-$10^5$ sec; here $P_{min} \sim 10^{-4}$ ergs/cm$^2\cdot$sec. In Bernshteyn's papers [6,7], the signal separated below the noise level was a monochromatic sinusoid. If the useful signal has a continuous spectrum, these calculations ought to be modified somewhat, but we shall not go into this question here, since these modifications are the same for the interferometer as for electromechanical experiments [1]. Technically, experiments with an interferometer to detect natural low-frequency gravitational waves are very complex. It is necessary to have stable apparatus, and the air must be evacuated along all the optical paths. Since the frequency, polarization, and direction of propagation of the wave are all unknown, it is necessary to have several interferometers and seek a correlation between them [1].

In conclusion the authors wish to express their gratitude to Professor V. L. Ginzburg for his valuable remarks and comments in connection with the present work.

Rec'd: 3 March 1962

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